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Nuclear Matrix Elements of Axial-Charge Exchange Currents Derived in Heavy-Fermion Chiral Perturbation Theory

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Abstract

We calculate shell-model matrix elements of the axial-charge exchange current operators that have been obtained up to the next-to-leading order from heavy-fermion chiral perturbation theory. It is found that loop corrections to the soft one-pion-exchange contribution are small (around 10 %) and have no significant dependence on the nuclear mass number or on the valence-nucleon orbits. These results render further support to the chiral-filtering conjecture.

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1 Introduction

The nuclear axial-charge operator, which carries valuable information about the role of the chiral symmetry in nuclear medium, has been the subject of a great deal of experimental and theoretical work (for reviews, see [1, 2]). The crucial point is that A^0, the time component of the axial current, is predicted to have a large exchange current due to a one-pion exchange diagram, the structure and strength of which are dictated by the soft-pion theorem [3]. According to a general argument by Kubodera, Delorme and Rho [3], the impulse-approximation (IA) one-body current A^0(1-body) should receive a large meson-exchange-current correction A^0(mec) which is mainly due to the soft-pion-exchange two-body current A^0(soft-π):

$$A^0 = A^0(\text{1-body}) + A^0(\text{mec}) \tag{1}$$

where

$$A^0(\text{1-body}) = \frac{g_A}{2m_{\rm N}}\,\vec{\sigma}\cdot\vec{p}\,\tau^{\pm}, \tag{2}$$

$$A^0(\text{mec}) \simeq A^0(\text{soft-}\pi) = \frac{g_A}{4{f_\pi}^2}\frac{m_\pi}{4\pi r}\left(1+\frac{1}{m_\pi r}\right)\mathrm{e}^{-m_\pi r}\;(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\hat{r}\,(\vec{\tau}_1\times\vec{\tau}_2)^{\pm}, \tag{3}$$

with $\vec{r}=\vec{r}_1-\vec{r}_2$. Here, $\vec{p}=\frac{1}{2}(\vec{p}_i+\vec{p}_f)$ with $\vec{p}_i$ and $\vec{p}_f$ the incoming and outgoing nucleon momentum; f_π is the pion-decay constant, g_A the axial-vector coupling constant, $m_{\rm N}$ the nucleon mass and m_π the pion mass. A useful measure of the typical effects of A^0(soft-π) was given by Delorme [4]. For a system of one valence nucleon added to the Fermi gas, Delorme derived the Hartree-Fock-type effective single-particle operator arising from A^0(soft-π), and evaluated its strength relative to the single-particle operator, eq.(2). This Fermi gas model estimate indicates that the original single-particle operator should be enhanced by $\sim 54\%$ ($\sim 39\%$) at nuclear matter density (half nuclear density). Thus, indeed, the two-body effect is very large. The replacement of the uncorrelated two-nucleon relative-motion wave functions used in the Fermi gas model with typical short-range-correlated wave functions would reduce the enhancement to some extent but, even with this reduction, the effect of A^0(soft-π) is still expected to be of a substantial magnitude.

All the existing experimental information supports the significant enhancement of A^0(1-body) due to the soft-pion exchange current (see the extensive bibliographies in [1, 2]). The most convincing evidence comes from Warburton *et al.*'s systematic analyses [5, 6] of the first-forbidden transitions over a wide range of the periodic table. The latest results of these analyses [6] can be summarized in terms of $\delta_{\rm mec}$, the ratio of the meson-exchange-current contribution to the IA one-body contribution:

$$\delta_{\rm mec} = \frac{\langle A^0(\text{mec})\rangle}{\langle A^0(\text{1-body})\rangle}.$$

The best fit to data gives:

$$\begin{aligned}\delta_{\rm mec}(^{16}{\rm O}) &= 0.61(3),\\ \delta_{\rm mec}(^{132}{\rm Sn}) &= 0.82(7),\\ \delta_{\rm mec}(^{208}{\rm Pb}) &= 0.79(4).\end{aligned} \tag{4}$$

These results on $\delta_{\rm mec}$ are certainly compatible with the sizeable contribution of $A^0(\text{soft-}\pi)$; in fact, the empirically determined enhancements even seem somewhat stronger than anticipated from the $A^0(\text{soft-}\pi)$ contribution.

Recent theoretical developments are concerned with: (i) the evaluation of heavy-meson exchange diagrams [7, 8]; (ii) the estimation of higher order terms in chiral perturbation [9, 10]; (iii) possible consequences of the in-medium scaling of particle masses [11].

All these developments are related to the chiral-filtering conjecture [3, 22], which states that, whenever there is no kinematical suppression, one-soft-pion exchange currents should dominate, with other shorter-ranged currents "filtered" off by the nuclear medium. Although a reasonably reliable argument for the dominance of $A^0(\text{soft-}\pi)$ in $A^0(\text{2-body})$ was presented in [3], the explicit demonstration of the suppression of other short-ranged contributions was left for future studies. The subsequent accumulation of experimental evidence in support of the chiral-filtering mechanism has further increased the importance of this explicit demonstration.

Several authors addressed this and related issues by evaluating the Feynman diagrams involving heavy-meson exchange [7, 8]. The upshot of this approach is that some of the short-range exchange currents can be rather large but that they tend to cancel among themselves. The net contribution of the heavy-meson exchange diagrams is to enhance $A^0(\text{soft-}\pi)$ by about the right amount to reproduce $\delta_{\rm mec}$. Although the idea of heavy-meson-exchange currents emerges naturally from the picture of the boson-exchange nuclear interactions and from the Dirac phenomenology [12, 13] and although its numerical results are generally encouraging, the convergence property of this approach is a non-trivial question. At present, there seems to be no clue to a relevant expansion parameter that controls a multitude of complicated diagrams describing short-range exchange currents. Also, the drastic cancellation between the individually large terms tends to make their net contribution quite model-dependent, a problem which awaits further investigations.

A systematic study of $A^0(\text{2-body})$ based on chiral perturbation theory (ChPT) was taken up by Rho [9] with the use of the heavy-fermion formalism (HFF) [14, 15, 16]. In ChPT, the chiral symmetry and well-defined counting rules allow a systematic construction of an effective Lagrangian. Furthermore, the use of HFF enables us to carry out a consistent perturbation expansion even for systems that involve the nucleons[#1]. A proof of the soft-pion-exchange dominance to leading order of chiral expansion was given by Rho [9]. The

[#1] From a formal point of view the general application of ChPT to a many-body system involves a subtlety, since the system has an extra scale $k_{\rm F}$ (Fermi momentum) in addition to the usual chiral-symmetry breaking

extension of the ChPT calculation to the next order (one-loop order) was carried out by Park *et al.* [10]. Park *et al.*'s calculation indicates that the loop correction to the soft-pion-exchange contribution is very small, supporting the robustness of the chiral filter mechanism. In HFF, baryon momentum-dependent terms appear as higher order interaction terms in the chiral counting, and this feature leads to a drastic reduction of the contributions of the heavy-meson exchange diagrams. The relation between the heavy-meson-exchange approach and ChPT is an interesting problem that warrants further careful studies.

In [10], the axial-charge transition matrix elements were calculated with the simple-minded Fermi-gas model. To obtain more reliable estimates of the contributions of higher-order terms in the chiral expansion, we need to use more realistic nuclear wave functions than those of the Fermi gas. Also, according to Towner [8], the matrix elements of the heavy-meson exchange currents exhibit significant dependence on the valence orbits. It is important to examine whether or not the loop corrections in ChPT shows a similar shell dependence. In this note, we calculate the shell-model matrix elements of the axial-charge exchange current operators obtained in [10] up to next-to-the-leading order in ChPT. Our results indicate that the loop corrections calculated in the shell model are as small as indicated by the Fermi gas model estimation. Furthermore, the mass- and state-dependence of the loop correction is negligible. These results give further support to the chiral-filtering conjecture. In the light of our new results, we shall also discuss the problem of the "extra" enhancement in the empirical $\delta_{\rm mec}$. In particular, we comment on the interrelation between the heavy-meson-exchange-current method [7, 8] and the ChPT with in-medium mass scaling [11].

2 Calculation

We begin by specifying our conventions and definitions. We introduce, as we did in [10], the quantity $\hat{\mathcal{M}}$, which is different from the axial-charge operator A^0 by just an overall factor,

$$\hat{\mathcal{M}} = \frac{4f_\pi^2}{g_A}\, A^0 = \hat{\mathcal{M}}_{\text{1-body}} + \hat{\mathcal{M}}_{\text{2-body}} + \cdots . \tag{5}$$

Using $\hat{\mathcal{M}}$ is slightly more convenient than using A^0 itself. In eq.(5), $\hat{\mathcal{M}}_{n\text{-body}}$ represents an n-body operator, and the ellipsis denotes $\hat{\mathcal{M}}_{n\text{-body}}$ ($n \geq 3$), which are ignored. $\hat{\mathcal{M}}_{\text{1-body}}$ is

scale $\Lambda\sim$1 GeV. However, it seems justifiable to use ChPT in the following limited context. We start with the generally accepted paradigm that the diluteness of nuclear matter allows us to concentrate on 1-body and 2-body responses to external probes, ignoring n-body ($n \geq 3$) contributions, and that the effective operators describing these responses can be obtained by considering Feynman diagrams involving one nucleon (two nucleons) for the 1-body (2-body) operators. Once a one-nucleon or two-nucleon subsystem is isolated from the A-body system, we use ChPT as a method to calculate the Feynman diagrams pertaining to these subsystems systematically and consistently with the basic chiral symmetry of QCD. In the present paper, the word "ChPT" should be understood in this context.

the impulse approximation term,

$$\hat{\mathcal{M}}_{\text{1-body}} = \frac{2f_\pi^2}{m_{\rm N}}\,\vec{\sigma}\cdot\vec{p}\,\tau^{\pm}. \tag{6}$$

The two-body exchange current, $\hat{\mathcal{M}}_{\text{2-body}}$, is decomposed into a tree part and a loop correction. The tree part is the seagull term considered in the original paper [3]. Meanwhile according to [10], the loop correction can be decomposed into a one-pion exchange part (a loop correction to the tree part through vertex renormalization) and a two-pion-exchange part :

$$\begin{aligned}\hat{\mathcal{M}}_{\text{2-body}} &= \hat{\mathcal{M}}_{\rm tree} + \hat{\mathcal{M}}_{\rm loop},\\ \hat{\mathcal{M}}_{\rm loop} &= \hat{\mathcal{M}}_{1\pi} + \hat{\mathcal{M}}_{2\pi},\end{aligned} \tag{7}$$

with

$$\begin{aligned}\hat{\mathcal{M}}_{\rm tree} &= \hat{\mathcal{T}}^{(1)}\, f_{\rm tree}(r)\\ \hat{\mathcal{M}}_{1\pi} &= \hat{\mathcal{T}}^{(1)}\, f_{1\pi}(r),\\ \hat{\mathcal{M}}_{2\pi} &= \hat{\mathcal{T}}^{(1)}\, f_{2\pi(1)}(r) + \hat{\mathcal{T}}^{(2)}\, f_{2\pi(2)}(r).\end{aligned} \tag{8}$$

The spin-isospin operators $\hat{\mathcal{T}}^{(1)}$ and $\hat{\mathcal{T}}^{(2)}$ are defined as

$$\begin{aligned}\hat{\mathcal{T}}^{(1)} &= (\vec{\tau}_1\times\vec{\tau}_2)^{\pm}\ \hat{r}\cdot(\vec{\sigma}_1+\vec{\sigma}_2),\\ \hat{\mathcal{T}}^{(2)} &= (\vec{\tau}_1+\vec{\tau}_2)^{\pm}\ \hat{r}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2),\end{aligned} \tag{9}$$

where $\hat{r}$ is the unit vector of $\vec{r}=\vec{r}_1-\vec{r}_2$ and r is the norm of $\vec{r}$. Functions $f_{\rm tree}(r)$, $f_{1\pi}(r)$ and $f_{2\pi(1,2)}(r)$ are given in [10],

$$f_{\rm tree}(r) = \frac{d}{dr}\left(-\frac{1}{4\pi r}\mathrm{e}^{-m_\pi r}\right), \tag{10}$$

$$\begin{aligned}f_{1\pi}(r) &= c_3^R\frac{m_\pi^2}{f_\pi^2}\, f_{\rm tree}(r)\\ &+ \frac{1}{16\pi^2 f_\pi^2}\frac{d}{dr}\left(-\frac{1+3g_A^2}{2}\left[K_0(r)-\tilde{K}_0(r)\right]+(2+4g_A^2)\left[K_2(r)-\tilde{K}_2(r)\right]\right),\end{aligned} \tag{11}$$

$$\begin{aligned}f_{2\pi(1)}(r) &= \frac{1}{16\pi^2 f_\pi^2}\frac{d}{dr}\left(-\frac{3g_A^2-2}{4}K_0(r)-\frac{1}{2}g_A^2K_1(r)\right),\\ f_{2\pi(2)}(r) &= \frac{1}{16\pi^2 f_\pi^2}\frac{d}{dr}\left(2g_A^2K_0(r)\right),\end{aligned} \tag{12}$$

The explicit forms of the functions $K_i(r)$ and $\tilde{K}_i(r)$ are given in Appendix B of ref.[10].

As noted in [10], the constant c_3^R can be extracted from the isovector Dirac form factor of the nucleon, *i.e.*,

$$c_3^R\frac{m_\pi^2}{f_\pi^2} = \frac{m_\pi^2}{6}\langle r^2\rangle_1^V \simeq 0.04784. \tag{13}$$

In shell-model calculations, it is convenient to write the above expressions in terms of spherical tensors [17],

$$
\begin{aligned}
i\hat{\mathcal{M}}_{\text{1-body}} &= -\sqrt{3}\,\frac{2f_\pi^2}{m_{\rm N}}\,\tau^\pm\,[\vec{\sigma},\ i\vec{p}]^{(0)}\,, \\
i\hat{\mathcal{M}}_i &= -\sqrt{6}\,\Sigma_m^{(1)}(\tau)\,\left[\hat{r},\,\Sigma_n^{(1)}(\sigma)\right]^{(0)}\ f_i(r)
\end{aligned}
\tag{14}
$$

where $(m,n)=(2,4)$ for $i=$ 'tree', $1\pi,\ 2\pi(1)$, which are proportional to $\hat{\mathcal{T}}^{(1)}$, and $(m,n)=(4,2)$ for $i=2\pi(2)$, which is proportional to $\hat{\mathcal{T}}^{(2)}$. Here, $\Sigma_m^{(1)}(\tau)$ and $\Sigma_n^{(1)}(\sigma)$ are defined in [17],

$$
\begin{aligned}
\Sigma_2^{(1)}(\tau) &= \frac{i}{\sqrt{2}}\,(\vec{\tau}_1\times\vec{\tau}_2)^\pm, \\
\Sigma_4^{(1)}(\tau) &= (\vec{\tau}_1+\vec{\tau}_2)^\pm
\end{aligned}
\tag{15}
$$

and similarly for $\Sigma_n^{(1)}(\sigma)$, $n=2,4$. The square bracket with superscript (0) in the above equations represents the vector coupling of two spherical tensors to a resultant[#2]; in this case two vectors are coupled to form a tensor of rank zero :

$$
\left[\vec{a},\,\vec{b}\right]^{(0)} = -\frac{1}{\sqrt{3}}\,\vec{a}\cdot\vec{b}. \tag{16}
$$

In the simplest version of the shell model, the ground state (Fock state) of a nucleus is taken as a Slater determinant of single-particle states, which here will be chosen as eigenstates of a spherical simple harmonic oscillator. In an oscillator Hamiltonian, there are two parameters, one is the mass, which will be identified as the nucleon mass, and the other is the oscillator frequency, ω, which is determined by

$$
\hbar\omega = \left(45\,A^{-1/3} - 25\,A^{-2/3}\right)\ [\text{MeV}] \tag{17}
$$

where A is the mass number of the nucleus. For an odd-mass nucleus, the wavefunction is written as a simple product of a Fock state for the neighbouring even-mass nucleus and a single-particle state, ψ_a, *viz.* $|\psi_a;F\rangle$. Here the ψ_a denotes all the quantum numbers of the single-particle states indexed by the subscript a, $\psi_a=(n_a,\ l_a,\ s_a=\frac{1}{2},\ j_a,\ j_{az},\ t_a=\frac{1}{2},\ t_{az})$, where $n_a(=0,1,2,\cdots)$ is the principal quantum number, l_a the orbital angular momentum, s_a the spin, j_a the total spin, and the subscript z identifies the third component. The use of such a shell-model calculation is given in detail in [17, 8]; here we just explain the definition of reduced matrix elements and some features of our calculation.

Consider an operator with definite multipolarity J and isospin T with third component J_z and T_z, $X^{(J,T)}_{J_z,T_z}$; our axial-charge operator can be regarded as $X^{(0,1)}_{0,T_z}$. Our goal is

[#2] In general,

$$
\left[Y^{(k_1)}(1),\ Y^{(k_2)}(2)\right]^{(K)}_M \equiv \sum_{m_1,m_2}\langle k_1m_1,k_2m_2|KM\rangle\,Y^{(k_1)}_{m_1}(1)\,Y^{(k_2)}_{m_2}(2)
$$

for any tensors $Y^{(k_1)}$ and $Y^{(k_2)}$.

to calculate the matrix element $\langle\psi_a;F|X^{(J,T)}_{J_z,T_z}|\psi_b;F\rangle$. For the case when X is a one-body operator, the result trivially reduces to a single-particle matrix element

$$\langle\psi_a|X^{(J,T)}_{J_z,T_z}|\psi_b\rangle \equiv \langle j_b j_{bz}, JJ_z|j_a j_{az}\rangle\langle t_b t_{bz}, TT_z|t_a t_{az}\rangle\langle a||X^{(J,T)}||b\rangle \tag{18}$$

where a denotes quantum numbers of states ψ_a without third components, $a=(n_a,l_a,j_a,s_a,t_a)$. This expression serves to define our conventions for reduced matrix elements. For a 2-body operator, X, the reduced matrix element, $\langle a;F||X^{(J,T)}||b;F\rangle$, is

$$\begin{aligned}\langle\psi_a;F|X^{(J,T)}_{J_z,T_z}|\psi_b;F\rangle &= \sum_{\psi_h\in F}\langle\psi_a,\psi_h|X^{(J,T)}_{J_z,T_z}|\psi_b,\psi_h\rangle_{\rm AS}\\ &\equiv \langle j_b j_{bz}, JJ_z|j_a j_{az}\rangle\langle t_b t_{bz}, TT_z|t_a t_{az}\rangle\langle a;F||X^{(J,T)}||b;F\rangle\end{aligned}$$

with

$$\begin{aligned}\langle a;F||X^{(J,T)}||b;F\rangle &= \sum_{h\in F}\sum_{T_1T_2}\sum_{J_1J_2}D(j_aj_bj_h,J_1J_2,J;F)D(t_at_bt_h,T_1T_2,T;F)\\ &\times\ \langle(ah)J_1T_1||X^{(J,T)}||(bh)J_2T_2\rangle_{\rm AS}\,,\end{aligned} \tag{19}$$

$$\langle(ah)J_1T_1||X^{(J,T)}||(bh)J_2T_2\rangle_{\rm AS} \equiv \frac{\langle a,h;J_1J_{1z},T_1T_{1z}|X^{(J,T)}_{J_z,T_z}|b,h;J_2J_{2z},T_2T_{2z}\rangle_{\rm AS}}{\langle J_2J_{2z},JJ_z|J_1J_{1z}\rangle\langle T_2T_{2z},TT_z|T_1T_{1z}\rangle} \tag{20}$$

and

$$D(j_aj_bj_h,J_1J_2,J;F)=\sum_{j_{hz}\in F}\sum_{J_{1z}J_{2z}}\frac{\langle j_aj_{az},j_hj_{hz}|J_1J_{1z}\rangle\langle j_bj_{bz},j_hj_{hz}|J_2J_{2z}\rangle\langle J_2J_{2z},JJ_z|J_1J_{1z}\rangle}{\langle j_bj_{bz},JJ_z|j_aj_{az}\rangle}. \tag{21}$$

A similar definition follows for $D(t_at_bt_h,T_1T_2,T;F)$. The subscript 'AS' implies the matrix element is antisymmetric under the permutation of the two-particle states. The difference between summation over $\psi_h\in F$ and that over $h\in F$ should be understood,

$$\sum_{\psi_h\in F}=\sum_{h\in F}\ \sum_{j_{hz}\in F}\ \sum_{t_{hz}\in F}.$$

When the neighbouring even-mass nucleus forms a closed shell, D becomes simpler :

$$D(j_aj_bj_h,J_1J_2,J;F)_{\rm closed}=\frac{\sqrt{2J_1+1}\,\sqrt{2J_2+1}}{\sqrt{2j_a+1}\,\sqrt{2j_b+1}}\,U(j_aj_h\,JJ_2;J_1j_b) \tag{22}$$

where the subscript 'closed' means that j_{hz} runs from $-j_h$ to j_h, and U is a recoupling coefficient defined in [17] and is related to the 6-j symbol. Now in our case, $J=0$, $T=1$, $t_a=t_b=t_h=\frac{1}{2}$, $s_a=s_b=s_h=\frac{1}{2}$, and we consider only closed shells in j-space[#3], in which case we have a much simpler expression,

$$D(j_aj_bj_h,J_1J_2,J=0;F)_{\rm closed}=\delta_{J_1,J_2}\,\delta_{j_a,j_b}\,\frac{2J_1+1}{2j_a+1}. \tag{23}$$

[#3] This is not the case for A=96, where there is one unclosed shell. Here we simply neglect contribution coming from this unclosed shell.

But in isospin space, we consider not only charge-symmetric $N = Z$ cases but also neutron-rich cases, and therefore we should retain the original definition,

$$\begin{aligned} D_T(T_1, T_2; F) &\equiv D(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}, T_1 T_2, T = 1; F) \\ &= \sum_{t_{hz} \in F} \sum_{T_{1z} T_{2z}} \frac{\langle \frac{1}{2} t_{az}, \frac{1}{2} t_{hz} | T_1 T_{1z} \rangle \langle \frac{1}{2} t_{bz}, \frac{1}{2} t_{hz} | J_2 J_{2z} \rangle \langle T_2 T_{2z}, 1 T_z | T_1 T_{1z} \rangle}{\langle \frac{1}{2} t_{bz}, 1 T_z | \frac{1}{2} t_{az} \rangle} \end{aligned} \tag{24}$$

With eqs.(19, 23, 24) here and eqs.(53-56) of ref.[17], it is straightforward to calculate the reduced matrix elements of the axial-charge operator.

3 Numerical Results

It is convenient to represent the results in terms of a ratio

$$\delta_x \equiv \frac{\langle \psi_a; F | \hat{\mathcal{M}}_x | \psi_b; F \rangle}{\langle \psi_a; F | \hat{\mathcal{M}}_{\text{1-body}} | \psi_b; F \rangle} = \frac{\langle a; F || \hat{\mathcal{M}}_x || b; F \rangle}{\langle a || \hat{\mathcal{M}}_{\text{1-body}} || b \rangle}, \tag{25}$$

where x = 'tree', 1π, 2π, 'loop' or '2-body'. Thus δ_x is the contribution of the x part relative to the 1-body contribution (impulse approximation): δ_{tree} is the ratio of the one-pion-exchange contribution evaluated in the soft-pion limit with no loop correction, $\delta_{1\pi}$ shows the form-factor effect of one-pion-exchange coming from the loop correction to the vertices, and $\delta_{2\pi}$ gives the two-pion-exchange contribution. The δ_{loop} stands for the total loop contribution compared to the impulse approximation:

$$\delta_{\text{loop}} = \delta_{1\pi} + \delta_{2\pi}, \tag{26}$$

while $\delta_{\text{2-body}}$, which is also denoted by δ_{mec}, represents the total 2-body contribution:

$$\delta_{\text{mec}} \equiv \delta_{\text{2-body}} = \delta_{\text{tree}} + \delta_{\text{loop}}. \tag{27}$$

Furthermore, we introduce η_{loop} as the ratio of the total loop contribution to the *tree* contribution:

$$\eta_{\text{loop}} \equiv \delta_{\text{loop}} / \delta_{\text{tree}} \tag{28}$$

In Table 1, we give two sets of parameters used in this calculation; "PMR" denotes parameters adopted in [10], and "OBEPR" denotes those obtained from the Bonn OBEPR potential [23]. There is a small difference of 1.1 % in the pion mass between the two. There is also a larger difference in the pion-decay constant f_π: In PMR, an experimental value is adopted, while in using OBEPR, f_π is taken from the Goldberger-Treiman relation and the fitted value of the πNN coupling constant of $g_{\pi NN} = 13.68353$. We observe this difference in f_π has a significant effect on δ_{mec} but a smaller effect on η_{loop}

For each parameter set, we have used three choices of short-range correlation functions; the first two are a simple step function with different cut-offs, $\hat{g}_i(r) = \theta(r - d_i)$,

Table 1: Parameters used in the present work

	PMR	OBEPR
f_π	93.0000 MeV	86.3982 MeV
m_π	139.5673 MeV	138.0300 MeV
$m_{\rm N}$	938.2796 MeV	938.2796 MeV
g_A	1.26	1.26
d_1	0.70693 fm	0.71480 fm
d_2	0.5 fm	0.5 fm
q_c	3.93 fm^{-1}	3.93 fm^{-1}

Table 2: Contributions to $\delta_{\rm mec}$ for various choices of parameters for the $1s_{1/2} \to 0p_{1/2}$ transition in A=16 : $\hat{g}_1(r) = \theta(r-d_1)$ with $d_1 = 0.5\ m_\pi^{-1} \simeq 0.71$ fm, $\hat{g}_2(r) = \theta(r-d_2)$ with $d_2 = 0.5$ fm and $\hat{g}_3(r) = 1-j_0(q_c r)$ with $q_c = 3.93$ fm^{-1}

	PMR			OBEPR		
$\hat{g}(r)$	$\hat{g}_1(r)$	$\hat{g}_2(r)$	$\hat{g}_3(r)$	$\hat{g}_1(r)$	$\hat{g}_2(r)$	$\hat{g}_3(r)$
$\delta_{\rm tree}$	0.407	0.459	0.480	0.472	0.535	0.560
$\delta_{1\pi}$	−0.027	−0.056	−0.080	−0.039	−0.080	−0.112
$\delta_{2\pi}$	0.068	0.120	0.160	0.091	0.162	0.216
$\delta_{\rm loop}$	0.041	0.063	0.080	0.052	0.081	0.104
$\delta_{\rm mec}$	0.449	0.522	0.560	0.524	0.616	0.663
$\eta_{\rm loop}$	10.2 %	13.8 %	16.6 %	11.0 %	15.2 %	18.5 %

$d_1 = \frac{1}{2}m_\pi^{-1} \simeq 0.71$ fm, $d_2 = 0.5$ fm. The third correlation function is $\hat{g}_3(r) = 1-j_0(q_c\, r)$, where $j_0(x) = \sin(x)/x$ and q_c is fixed as $q_c = 3.93$ fm^{-1} [18].

In Table 2, we show the contributions to $\delta_{\rm mec}$ for the $1s_{1/2} \to 0p_{1/2}$ transition in A=16 with various choices of parameters and correlation functions. Successive rows in the table give the soft-pion tree contribution, the 1π-loop correction, the 2π-loop correction, the total loop correction (sum of rows 4 and 5), and finally the total meson-exchange contribution (sum of rows 3, 4 and 5). The last row denoted by $\eta_{\rm loop}$ expresses the total loop correction relative to the 'tree' contribution. This correction is generally small ranging from 10% for a cut-off of $d \simeq 0.7$ fm to 15% for $d = 0.5$ fm.

A short-range (SR) correlation function is necessary for calculations such as these, but there is little guidance as to its choice. Inevitably therefore some model dependence is injected here representing short-range phenomenology that is under poor control. For long-range operators, such as those arising from pion-range tree graphs, this is not a serious

problem as the matrix elements are not strongly influenced by the choice of correlation function. For the operators from the shorter ranged loop graphs, however, we can get a factor of two difference depending on the choice. This is evident from the results given in Table 2. Any conclusions, therefore, have to be tempered by this reality.

A SR correlation function is required in this work for two reasons, and a common function is used here to cover both. First, there are correlations in the nuclear physics many-body problem. Because the two-body operators in eq. (8) are expressed in terms of the relative separation of two nucleons, $r = |\vec{r}_1 - \vec{r}_2|$, it is convenient if the shell-model wavefunctions are expressed in terms of similar co-ordinates. This is quite practicable if the shell-model Hamiltonian is the harmonic oscillator. Then the coefficients of transformation from a single-particle basis to relative and center-of-mass basis are known. However, there is one drawback to the harmonic oscillator Hamiltonian: its eigenfunctions are not the eigenfunctions of the nucleon-nucleon interaction. This is particularly important in the relative coordinate, where the nucleon-nucleon interaction is known to have a strong short-range repulsion that makes the relative wavefunction go rapidly to zero as $r \to 0$, more rapidly than given by uncorrelated oscillator functions. Thus to incorporate this piece of many-body physics it is quite common to modify two-body operators by multiplying them by a SR correlation function, $\hat{g}(r)$, where $\hat{g}(r)$ is some function that tends to zero as $r \to 0$ and tends to unity for large r.

Second, in ChPT, loop integrations introduce high-momentum components. Although these loop integrals have been regularized and are finite, there are numerical difficulties in the Fourier transform to coordinate space. One way to deal with this is to impose a high-momentum cut-off. Another way is to impose a SR correlation function in co-ordinate space that would kill or ameliorate divergences at the origin. Consider a loop integral of order L. From the counting rules of ChPT and the nature of the loop integral, the most divergent piece has a form

$$r^k \left(\frac{\ln r}{r^2} \right)^L$$

where k is a non-negative integer depending on the process. In our case, $k = 1$ and $L = 1$. But, it is natural to assume that the correlation function is universal for all L. Thus, in order to ameliorate the divergence in all orders, we should have

$$\lim_{r \to 0} r^k \left(\frac{\ln r}{r^2} \right)^L \hat{g}(r) = 0 \text{ or finite}$$

for any finite L. This is a rather severe restriction. In practice, because we are interested in one-loop accuracy, this restriction can be milder. In our case, we should have, at least,

$$\lim_{r \to 0} \frac{\ln r}{r} \hat{g}(r) = 0 \text{ or finite.}$$

The step function $\theta(r-d)$ is the simplest SR correlation function and its underlying assumption for ChPT is that a nucleon has a hard core of radius d. It certainly satisfies all

Table 3: Contributions to $\delta_{\rm mec}$ with PMR parameters and with $\hat{g}(r) = \hat{g}_1(r)$

	A=16	A=40	A=48	A=96	A=132	A=208	A=208
$\hbar\omega$(MeV)	13.921	11.021	10.489	8.635	7.874	6.883	6.883
initial	$1s_{1/2}$	$1p_{3/2}$	$1p_{3/2}$	$2s_{1/2}$	$1f_{7/2}$	$1g_{9/2}$	$2p_{1/2}$
final	$0p_{1/2}$	$0d_{3/2}$	$0d_{3/2}$	$1p_{1/2}$	$0g_{7/2}$	$0h_{9/2}$	$2s_{1/2}$
$\delta_{\rm tree}$	0.407	0.435	0.523	0.441	0.554	0.553	0.462
$\delta_{1\pi}$	−0.027	−0.031	−0.032	−0.028	−0.034	−0.035	−0.026
$\delta_{2\pi}$	0.068	0.077	0.073	0.069	0.080	0.084	0.066
$\delta_{\rm loop}$	0.041	0.046	0.040	0.041	0.046	0.049	0.040
$\delta_{\rm mec}$	0.449	0.481	0.564	0.481	0.600	0.602	0.502
$\eta_{\rm loop}$	10.2 %	10.5 %	7.7 %	9.3 %	8.3 %	8.8 %	8.6 %

the required properties stated above, it is easy to implement and it has a very clear physical interpretation. Similarly a step function has been used in the nuclear many-body problem as well [20], where the interpretation of d now relates to the range of the short-range repulsion in the nucleon-nucleon interaction.

There are other choices of SR correlation functions in use in nuclear physics. For example, Brown *et al.* [18] choose a form $1 - j_0(q_c r)$, with $q_c = 3.93\text{fm}^{-1}$, where q_c has been adjusted to reproduce the dominant Fourier components of a realistic two-body correlation function calculated with the Reid soft-core potential. Note that the value of q_c matches the inverse of the Compton wavelength for the ω-meson. Thus this form of correlation function is particularly appropriate for any potential, such as the Bonn potential [23], where the short-range repulsion is generated by 'enhanced' meson-nucleon coupling for the ω-meson. The Bonn potential is 'softer' than the historic hard-core potentials and this correlation function is likewise softer than the form $\theta(r-d)$. If we define an effective cut-off, $d_{\rm eff}$, for $\hat{g}_3(r) = 1 - j_0(q_c r)$, such that the correlation function $\theta(r - d_{\rm eff})$ gives approximately the same result as $\hat{g}_3(r)$, then we obtain

$$d_{\rm eff} \simeq 0.4 \text{ fm}. \tag{29}$$

This value is smaller than the choices of $d_1 \simeq 0.7$ fm and $d_2 = 0.5$ fm used with $\hat{g}_1(r)$ and $\hat{g}_2(r)$, respectively.

The problem with the correlation function $1 - j_0(q_c r)$ is that it does not meet the requirements from ChPT of ameliorating the divergences at the origin. In particular it could not be used for two-loop or higher calculations. Thus in the following discussions we will give less weight to calculations using $\hat{g}_3(r)$.

In Table 3, we display the contributions to $\delta_{\rm mec}$ for various odd-mass nuclei of closed-shell-plus-one configuration, where the mass number of the closed shell, A, is indicated.

The purpose is to investigate the mass dependence of the two-body contributions. Here, the PMR parameter set with $\hat{g}_1(r)$ is adopted. In all cases, the 1π-loop and 2π-loop corrections are small and of opposite sign so that the resultant $\delta_{\rm loop}$ is small. Further, when expressed relative to the 'tree' contribution, the $\eta_{\rm loop}$ shows little mass or state dependence. We will discuss mass dependence further in the next section.

In Table 4, we present our results for $\delta_{\rm mec}$ and $\eta_{\rm loop}$ for various single-particle transitions in a number of different nuclei ranging from light to heavy, with the PMR parameters and the $\hat{g}_1(r)$ correlation function.

Table 4: Calculations of $\delta_{\rm mec}$ and $\eta_{\rm loop}$ for a range of transitions in various nuclei with the PMR parameters and including the short-range correlation function, $\hat{g}(r) = \hat{g}_1(r) = \theta(r - d_1)$ with $d_1 = 0.5\ m_\pi^{-1} \simeq 0.7$ fm

A=16, $\hbar\omega = 13.921$			A=40, $\hbar\omega = 11.021$		
Transition	$\delta_{\rm mec}$	$\eta_{\rm loop}$	Transition	$\delta_{\rm mec}$	$\eta_{\rm loop}$
$0d_{3/2} \to 0p_{3/2}$	0.351	9.2 %	$0f_{5/2} \to 0d_{5/2}$	0.350	9.4 %
$1s_{1/2} \to 0p_{1/2}$	0.449	10.2 %	$1p_{3/2} \to 0d_{3/2}$	0.481	10.5 %
$0p_{1/2} \to 0s_{1/2}$	0.508	9.3 %	$1p_{1/2} \to 1s_{1/2}$	0.389	10.1 %
			$0d_{3/2} \to 0p_{3/2}$	0.475	9.4 %
			$1s_{1/2} \to 0p_{1/2}$	0.536	10.2 %
A=48, $\hbar\omega = 10.489$			A=96, $\hbar\omega = 8.635$		
$0f_{5/2} \to 0d_{5/2}$	0.409	8.4 %	$0g_{7/2} \to 0f_{7/2}$	0.386	8.8 %
$1p_{3/2} \to 0d_{3/2}$	0.564	7.7 %	$1d_{5/2} \to 0f_{5/2}$	0.568	8.4 %
$1p_{1/2} \to 1s_{1/2}$	0.434	8.8 %	$1d_{3/2} \to 1p_{3/2}$	0.400	9.3 %
$0d_{3/2} \to 0p_{3/2}$	0.507	9.4 %	$2s_{1/2} \to 1p_{1/2}$	0.481	9.3 %
$1s_{1/2} \to 0p_{1/2}$	0.600	8.4 %	$0f_{5/2} \to 0d_{5/2}$	0.468	9.4 %
			$1p_{3/2} \to 0d_{3/2}$	0.593	8.7 %
			$1p_{1/2} \to 1s_{1/2}$	0.502	9.2 %
A=132, $\hbar\omega = 7.874$			A=208, $\hbar\omega = 6.883$		
$0h_{9/2} \to 0g_{9/2}$	0.400	7.8 %	$0i_{11/2} \to 0h_{11/2}$	0.390	8.2 %
$1f_{7/2} \to 0g_{7/2}$	0.600	8.3 %	$1g_{9/2} \to 0h_{9/2}$	0.602	8.8 %
$1f_{5/2} \to 1d_{5/2}$	0.406	7.7 %	$1g_{7/2} \to 1f_{7/2}$	0.395	8.1 %
$2p_{3/2} \to 1d_{3/2}$	0.517	8.3 %	$2d_{5/2} \to 1f_{5/2}$	0.530	8.8 %
$2p_{1/2} \to 2s_{1/2}$	0.437	8.1 %	$2d_{3/2} \to 2p_{3/2}$	0.418	8.5 %
$0g_{7/2} \to 0f_{7/2}$	0.482	8.3 %	$3s_{1/2} \to 2p_{1/2}$	0.476	8.8 %
$1d_{5/2} \to 0f_{5/2}$	0.610	8.9 %	$0h_{9/2} \to 0g_{9/2}$	0.461	8.5 %
$1d_{3/2} \to 1p_{3/2}$	0.501	8.1 %	$1f_{7/2} \to 0g_{7/2}$	0.608	9.3 %
$2s_{1/2} \to 1p_{1/2}$	0.559	8.4 %	$1f_{5/2} \to 1d_{5/2}$	0.475	8.4 %
			$2p_{3/2} \to 1d_{3/2}$	0.561	8.9 %
			$2p_{1/2} \to 2s_{1/2}$	0.502	8.6 %

From these tables, two conclusions emerge:

- The loop correction ($\eta_{\rm loop}$) is around 10 % with some dependence on the choice of SR correlation function. This indicates quite a small correction, confirming the dominance of the soft-pion-tree graph and the conclusions made in [10], where a simple-minded Fermi-gas model was adopted. The chiral filtering conjecture appears to hold.

- The loop correction ($\eta_{\rm loop}$) is essentially nuclear-mass and state independent. This is easily understood in that the tensor structure is common to the 'tree', 1π and part of the 2π operators displayed in eq.(8). These operators only differ in their radial functions. Furthermore in light nuclei with LS closed shells, the other part of the 2π operator also has the same tensor structure because the matrix element of $(\vec{\sigma}_1+\vec{\sigma}_2)\,(\vec{\tau}_1\times\vec{\tau}_2)$ is the same as that of $(\vec{\sigma}_1\times\vec{\sigma}_2)\,(\vec{\tau}_1+\vec{\tau}_2)$ in this case. From Tables 3 and 4, we observe that $\eta_{\rm loop}$ of neutron-rich nuclei is smaller than that of $N=Z$ nuclei by 10 or 20 %, which may be viewed as the difference in the matrix element of the two operators for neutron-excess orbitals.

4 Discussion

We should like to conclude with a more detailed discussion of the mass dependence in the results. To this end it is convenient to define a ratio, r, where

$$r = \frac{\delta_{\rm mec}(^{208}{\rm Pb})}{\delta_{\rm mec}(^{16}{\rm O})}. \tag{30}$$

Here $\delta_{\rm mec}$ is the correction expressed as a fraction of the 1-body impulse-approximation matrix element. For light nuclei in the vicinity of the closed shell, A=16, we consider the transition, $1{\rm s}_{1/2} \rightarrow 0{\rm p}_{1/2}$, that dominates the spectroscopy in this mass region, while for heavy nuclei in the vicinity of the closed shell, A=208, we consider the $1{\rm g}_{9/2} \rightarrow 0{\rm h}_{9/2}$ transition. This ratio was introduced by Towner [8], because it is rather insensitive to the choice of SR correlation functions and the parameters used. Since this choice represents one of the biggest uncertainties in the present work, the ratio, r, is an attractive quantity to discuss.

There is some experimental information on the ratio, r. It derives principally from experimental data on first-forbidden beta decays, as analyzed in the shell model by Warburton *et al.* [6]. The method is to compute beta-decay matrix elements in impulse approximation with the best available shell-model wavefunctions, and then allow the matrix element of the time-like part of the axial current to be multiplied by an enhancement factor, $\epsilon_{\rm mec}$. Note that

$$\epsilon_{\rm mec} = \frac{\langle \hat{\mathcal{M}}_{\text{1-body}}\rangle + \langle \hat{\mathcal{M}}_{\text{2-body}}\rangle}{\langle \hat{\mathcal{M}}_{\text{1-body}}\rangle} = 1+\delta_{\rm mec}. \tag{31}$$

A value for $\epsilon_{\rm mec}$ is obtained from a fit between experiment and calculation over a number of transitions in the mass region under study. The principal difficulty in this analysis is the

Table 5: Table of r-values. The asterisks on the entries are a reminder of the special meaning of $\delta_{\rm mec}$, explained in the text

	$\delta_{\rm mec}(^{16}{\rm O})$	$\delta_{\rm mec}(^{208}{\rm Pb})$	r	comments
tree	0.407	0.553	1.36	table 3 line 1, present work
loop	0.041	0.049	1.20	table 3 line 4, present work
mec	0.449	0.602	1.34	table 3 line 5, present work
tree	0.560	0.759	1.36	table 2, Towner [8]
correction	0.031	0.055	1.77	table 2, Towner [8]
mec	0.591	0.814	1.38	table 2, Towner [8]
BR scaling I	0.567*	0.885*	1.56	Kubodera-Rho [11, 25]
BR scaling II	0.595*	0.885*	1.49	
Experiment	0.61(3)	0.79(4)	1.30(9)	weak tensor force, Warburton [6]
Experiment	0.61(3)	0.91(5)	1.49(10)	strong tensor force, Warburton [6]

inevitable truncation required in the model space used in the particular shell-model calculation. Thus Warburton computes a correction for model-space truncations to first order in perturbation theory. This correction, however, is dependent on the choice of residual interactions used in the calculation. In particular, it depends quite sensitively on the strength of its tensor component. For weak tensor forces, such as obtained in the Bonn interaction [23], the value of r is 1.30 ± 0.09, while for strong tensor forces, such as obtained with the Paris potential [24], the value is 1.49 ± 0.10. In either case there is more enhancement of the axial-charge matrix element required in the lead region than in the oxygen region.

In Table 5 we list some calculated r-values from the present work and other sources [8, 11]. The first row gives just the one-pion tree-graph contribution, where a value of $r = 1.36$ indicates that most of the experimental mass dependence is accommodated by the shell model with just the soft-pion 2-body operator. The mass dependence arises because the number of core orbitals being summed over in eq.(19) is changing from light to heavy nuclei and the oscillator frequency parameter is reducing to reflect the increase in nuclear size. One can easily get similar effects in Fermi gas models. For example, Delorme [4] was the first to write down the expression for $\delta_{\rm tree}$ for a nucleon in a Fermi gas

$$\delta_{\rm tree} = \frac{m_{\rm N} k_{\rm F}}{4\pi^2 f_\pi^2} \left[1 + 2x - 2x(1+x) \, \ln\!\left(1 + \frac{1}{x}\right) \right] \qquad (32)$$

where $x = m_\pi^2/(4k_{\rm F}^2)$ and $k_{\rm F}$ is the Fermi momentum. In Fermi gas models, $k_{\rm F}$ is related to the nuclear density, $\rho = 2k_{\rm F}^3/(3\pi^2)$. If it is assumed that a valence nucleon in a light nucleus such as oxygen experiences only one half the nuclear matter density, then the appropriate

value of $k_{\rm F}$ to use in eq.(32) is $k_{\rm F} = 1.08$ fm^{-1}. For a valence nucleon in a heavy nucleus such as lead the appropriate nuclear matter value of $k_{\rm F} = 1.36$ fm^{-1} would be used. Then, with the PMR parameter set, $\delta_{\rm mec}(^{208}{\rm Pb}) = 0.54$ and $\delta_{\rm mec}(^{16}{\rm O}) = 0.39$ and an r-value of $r = 1.4$ is trivially obtained.[#4]

A similar idea has been discussed by Kubodera and Rho [11]. They argue from [21] that incorporation of approximate chiral and scale invariances of QCD leads to a chiral Lagrangian of low-energy hadrons in which the pion-decay constant and the hadron masses scale universally as a function of the matter density ρ according to:

$$\frac{m_{\rm N}^*}{m_{\rm N}} \approx \frac{m_\sigma^*}{m_\sigma} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\omega^*}{m_\omega} \approx \frac{f_\pi^*}{f_\pi} \equiv \Phi(\rho) \tag{33}$$

while the axial-coupling constant does not scale,

$$\frac{g_A^*}{g_A} \approx 1. \tag{34}$$

Here the asterisk refers to a value in a nuclear medium as opposed to the free hadron value. Note that the pion-decay constant, f_π, and the heavy meson masses all scale according to the same function. However the mass of the pion, being a Goldstone boson, is assumed not to scale, $m_\pi^* \approx m_\pi$. These assumptions are referred to as Brown-Rho (BR) scaling. But there is a subtlety explained in detail in Rho's lecture note [25]: the Gamow-Teller coupling constant g_A scales in medium not due to the above BR scaling but due to the short-range interactions between baryons; as a result, *the constant g_A^*/f_π^* associated with a pion exchange remains constant*,

$$\frac{g_A^*}{f_\pi^*} \approx \frac{g_A}{f_\pi}. \tag{35}$$

These considerations lead to

$$\langle A^0(\text{1-body})\rangle^* = \Phi(\rho)^{-1}\,\langle A^0(\text{1-body})\rangle, \tag{36}$$

and

$$\langle A^0(\text{mec})\rangle^* = \Phi(\rho)^{-1}\,\langle A^0(\text{mec})\rangle, \tag{37}$$

where the asterisked quantities should be evaluated with the scaled parameters defined in eqs.(33, 34, 35), while un-asterisked quantities should be evaluated with the parameters in free space, given in Table 1. Once the 1-body operator becomes ρ-dependent, we need to elaborate on the definition of $\epsilon_{\rm mec}$ [eq.(31)] by specifying what 1-body matrix element is used in the denominator. To be consistent with the way in which the empirical $\epsilon_{\rm mec}$ was deduced by Warburton *et al.* [5, 6], we must adopt the definition

$$\epsilon_{\rm mec} \equiv \frac{\langle A^0(\text{1-body})\rangle^* + \langle A^0(\text{mec})\rangle^*}{\langle A^0(\text{1-body})\rangle}. \tag{38}$$

[#4] With the OBEPR parameter set, these values become $\delta_{\rm mec}(^{208}{\rm Pb}) = 0.63$, $\delta_{\rm mec}(^{16}{\rm O}) = 0.45$ and $r = 1.4$. The r-value is insensitive to the parameter set used, which can be understood by noting that the r-value depends only on the value of m_π.

The $\delta_{\rm mec}$ which should appear in eq.(30) and which for the sake of clarity is denoted here by $\delta_{\rm mec}^{\rm Warb}$ is *numerically* defined by

$$\epsilon_{\rm mec} \equiv 1 + \delta_{\rm mec}^{\rm Warb}. \tag{39}$$

Meanwhile, eqs. (36) and (37) imply that

$$\epsilon_{\rm mec} = \Phi(\rho)^{-1} \left[1 + \frac{\langle A^0({\rm mec}) \rangle}{\langle A^0(\text{1-body}) \rangle} \right]. \tag{40}$$

Reinterpreting $\delta_{\rm mec}$ of eq.(25) in the present context, we identify

$$\frac{\langle A^0({\rm mec}) \rangle}{\langle A^0(\text{1-body}) \rangle} = \delta_{\rm mec}. \tag{41}$$

Then

$$\delta_{\rm mec}^{\rm Warb} = \Phi(\rho)^{-1} (1 + \delta_{\rm mec}) - 1. \tag{42}$$

We assume $\rho(^{208}{\rm Pb}) = \rho_0$ with $\rho_0 \simeq 0.17$ fm^{-3} being the normal nuclear matter density. There is latitude in choosing $\rho(^{16}{\rm O})$, and we consider here two cases. In Case I we assume, as previously, that $\rho(^{16}{\rm O}) = \rho_0/2$. To illustrate the sensitivity of the results to ρ, we also consider Case II, in which $\rho(^{16}{\rm O}) = 0.6\rho_0$. A typical choice of Φ is $\Phi(\rho) = 1 - 0.15\,(\rho/\rho_0)$. Then, using $\delta_{\rm mec}$ given in Table 3, we obtain, for Case I, $\delta_{\rm mec}^{\rm Warb}(^{16}{\rm O}) = 0.567$ and $\delta_{\rm mec}^{\rm Warb}(^{208}{\rm Pb}) = 0.885$, which leads to $r = 1.56$. The corresponding numbers for Case II are: $\delta_{\rm mec}^{\rm Warb}(^{16}{\rm O}) = 0.595$, $\delta_{\rm mec}^{\rm Warb}(^{208}{\rm Pb}) = 0.885$, and $r = 1.49$. These results are listed in Table 5 under the heading 'BR scaling'. We note that in this approach the scaling factor $\Phi(\rho)$ is the principal mechanism for explaining the observed mass dependence.

If we repeat the same procedure using $\delta_{\rm tree}$ instead of $\delta_{\rm mec}$, the resulting r-value will be somewhat larger; $r_{\rm tree} = 1.59$ for Case I, and $r_{\rm tree} = 1.51$ for Case II. As can be seen in Table 5, the corrections coming from pion loops evaluated in ChPT are almost nuclear mass independent, and this explains why the sum of tree graphs plus corrections leads to a smaller r-value than the tree graphs alone. This feature is in contrast to the results obtained by Towner [8] in a quite different approach. There the tree graphs are not evaluated in the soft-pion limit but are evaluated with full momentum dependence retained, including vertex form factors[#5]. This calculation, called the 'hard-pion approach', reduces the value of the tree graph significantly. The reduction, however, is largely compensated by heavy-meson pair graphs, which also give a sizeable contribution as pointed out by Kirchbach, Riska and Tsushima [7]. Thus the correction in this approach is computed from 'hard pions' plus 'heavy mesons' minus 'soft pions', and is given in line 5 of Table 5. This correction has significant mass dependence, such that the r-value of the sum of soft-pion tree graph plus corrections increases relative to soft pions alone.

[#5] We remark that form factors built into the Lagrangian have no place in ChPT; the momentum dependence of vertices in ChPT arises from radiative corrections, as one calculates higher-order diagrams in chiral expansion.

These two viewpoints, however, can be reconciled. When heavy-meson pair graphs are explicitly computed, the largest contributions come from σ and ω mesons. Mathematically the σ-meson contribution takes the form of the 1-body impulse approximation with the nucleon mass replaced by an effective mass. This was shown by Delorme and Towner [13] in a different context. Recently Birse [26] finds a similar result in the non-topological soliton model for a nucleon embedded in mean scalar and vector fields. This implies that the phenomenological σ-meson and the BR scaling play the same role, although their chiral properties are quite different. At this place, it should be noted that there is no way to introduce a σ-field in a theory where chiral symmetry is non-linearly realized. The only possible way is to define a chiral-scalar (and of course Lorentz scalar) field as Brown and Rho did in their BR scaling [21]. Therefore to make comparisons with ref.[8], BR scaling has to be added to the ChPT results.

In conclusion, we find that corrections to the soft-pion tree graph computed from 1-π and 2-π loop graphs in ChPT are small and around 10 %. This lends further support to the chiral filtering conjecture [3, 22]. Second, the mass dependence or density dependence evident in the analysis of the experimental data by Warburton *et al.* [6] has a variety of interlinking explanations ranging from the trivial mass dependence inherent in the shell model to the more fundamental role of heavy mesons or BR scaling.

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References

[1] I.S. Towner, Ann. Rev. Nucl. Part. Sci. **36** (1986) 115.

[2] K. Kubodera, Hyperf. Interact. **98** (1993) 3.

[3] K. Kubodera, J. Delorme and M. Rho, Phys. Rev. Lett.**40** (1978) 755.

[4] J. Delorme, Nucl. Phys. **A374** (1982) 541c.

[5] E.K. Warburton, Phys. Rev. Lett.**66** (1991) 1823; Phys. Rev. **C44** (1991) 233, 1024

[6] E.K. Warburton, I.S. Towner and B.A. Brown, Phys. Rev. **C49** (1994) 824;
E.K. Warburton and I.S. Towner, Phys. Lett. **B294** (1992) 1;
E.K. Warburton and I.S. Towner, Phys. Reports, to be published.

[7] M. Kirchbach, D.O. Riska and K. Tsushima, Nucl. Phys. **A542** (1992) 14.

[8] I.S. Towner, Nucl. Phys.**A542** (1992) 631.

[9] M. Rho, Phys. Rev. Lett. **66** (1991) 1275.

[10] T.-S. Park, D.-P. Min and M. Rho, Phys. Reports **233** (1993) 341.

[11] K. Kubodera and M. Rho, Phys. Rev. Lett. **67** (1991) 3479.

[12] J.D. Walecka, Ann. Phys. **83** (1974) 491; B.D.Serot and J.D. Walecka, Adv. Nucl. Phys. **16** (1986) 1.

[13] J. Delorme and I.S. Towner, Nucl. Phys. **A475** (1992) 720.

[14] S. Weinberg, Phys. Lett. **B251** (1990) 288; S. Weinberg, Nucl. Phys. **B363** (1991) 2.

[15] E. Jenkins and A. Manohar, Phys. Lett. **B255** (1991) 562; E. Jenkins, Nucl. Phys. **B368** (1991) 190.

[16] H. Georgi, Phys. Lett. **B240** (1990) 447.

[17] I.S. Towner, Nucl. Phys. **A540** (1992) 478.

[18] G.E. Brown, S.-O. Bäckman, E. Oset and W. Weise, Nucl. Phys.**A286** (1977) 191.

[19] G.A. Miller and J.E. Spencer, Ann. Phys. **100** (1976) 191.

[20] I.S. Towner, Phys. Reports **155** (1987) 263.

[21] G.E. Brown and M. Rho, Phys. Rev. Lett. **66** (1991) 2720.

[22] M. Rho and G.E. Brown, Comm. Part. Nucl. Phys. **10** (1981) 201.

[23] R. Machleidt, K. Holinde and Ch. Elster, Phys. Reports **149** (1987) 1.

[24] M. Lacombe *et al.*, Phys. Rev. **C21** (1980) 861.

[25] M. Rho, Saclay preprint, SPhT T93/110.

[26] M. C. Birse, Phys. Lett. **B316** (1993) 472.